

MATH 201B FALL 2009 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.*

- (1) Let P and Q be statements. Show that

$$(P \rightarrow Q) \wedge (Q \rightarrow P) \equiv (P \vee Q) \rightarrow (P \wedge Q).$$

- (2) Let A , B , and C be subsets of a set U . Prove that

$$(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C).$$

- (3) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.

- (4) Let n be a positive integer. Prove or disprove:

$$3 \mid (2^{2n} - 1).$$

- (5) Prove that $2^n > n^3$ for integers $n \geq 10$.

- (6) Give an ε - δ type proof that

$$\lim_{x \rightarrow 2} x^4 = 16.$$