

MATH 201B FALL 2009 PRACTICE TEST #1

Write clearly, on separate paper. All questions carry equal weight. You will receive credit for your three best answers.

(1) Show that $\log_{\sqrt{2}} 5$ is irrational.

(2) Prove

$$\forall 0 < x \in \mathbb{R}, \forall 1 < n \in \mathbb{Z}, (1+x)^n > 1+nx$$

by induction on n .

(3) Prove or disprove:

$$\forall n \in \mathbb{N}, 3^n > n^2.$$

(4) Prove or disprove:

$$\forall 1 < n \in \mathbb{Z}, 5 \mid (4^n - 1).$$