

MATH 201B FALL 2007 PRACTICE FINAL

*Write clearly. Credit is given for careful work.
All questions carry equal weight.*

1. Prove that

$$\sqrt{(x + y)^2} \leq \sqrt{x^2} + \sqrt{y^2}$$

for all real numbers x and y .

2. Prove that every nonempty inductive subset of \mathbb{N} is infinite.

3. Prove that

$$2 + 4 + 6 + \cdots + (2n - 2) + 2n = n^2 + n$$

for all natural numbers n .

4. Show that the sequence

$$a_n = n^2 \sin\left(\frac{n\pi}{2}\right)$$

does not tend to infinity as n tends to infinity.

5. Let θ be a real number. Prove that the sequence

$$a_n = \frac{\cos n\theta}{n}$$

tends to zero as n tends to infinity.

6. Let r and s be real numbers. Suppose that r is rational and s is irrational. Prove that $r - s$ is irrational.