MATH 201B FALL 2007 PRACTICE TEST #1

Write clearly, on separate paper. All questions carry equal weight.

(1) Write a careful proof of the following statement:
   The product of an even integer with an odd integer is even.

(2) The logical connective **xor** of propositions P and Q is defined by

$$P \textbf{ xor } Q = (P \vee Q) \wedge \neg(P \wedge Q)\,.$$

Write a truth table for P **xor** Q.

(3) Write out each of the following sets in mathematical notation. Use set-builder notation for the infinite sets, and the roster method for the finite sets.
   (a) The set of negative integers.
   (b) The set of integers whose square is less than 7.
   (c) The set of integers whose square is greater than 7.

(4) Consider the following sentences. If the sentence is a statement, say whether the statement is TRUE or FALSE. If the sentence is not a statement, say NOT A STATEMENT. (While no justification is required, there will be no partial credit for incorrect answers.)
   (a) $\exists\, m \in \mathbb{N}\,.\ m + n = 7$.
   (b) $\exists\, m \in \mathbb{N}\,.\ m + 7 = 5$.
   (c) $\exists\, m \in \mathbb{N}\,.\ m + 5 = 7$.

(5) Write out the negation of each of the following statements in formal symbolic notation (using quantifiers, standard notation for sets, etc.)
   (a) $\forall\, x \in \mathbb{R}\,,\ \exists\, y \in \mathbb{R}\,.\ x + y = 27$.
   (b) For real values of x, the quantity $x^2 + 6$ exceeds 5.
   (c) $\exists\, x \in \mathbb{R}\,.\ \forall\, y \in \mathbb{R}\,,\ \exists\, z \in \mathbb{R}\,.\ x \cdot y = z$.