MATH 2010 FALL 2025 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] Show that the set $E = \{q \in \mathbb{Q} \mid q^2 < 2\}$ has no supremum in the totally ordered set $(\mathbb{Q}, <)$.

(2) [5pts.] Consider a real number x with $-1 < x$. Prove that

$$(1+x)^r \geq 1 + rx$$

for $r \in \mathbb{N}$.

(3) [4pts.] Suppose that R is a positive real number. Consider a polynomial $p\colon [-R, R] \to \mathbb{R}$ with

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$$

for $a_n, \ldots, a_0 \in \mathbb{R}$. Show that $|p(x)|$ is bounded by

$$|a_n| \cdot R^n + |a_{n-1}| \cdot R^{n-1} + \cdots + |a_1| \cdot R + |a_0|$$

for $x \in [-R, R]$.