MATH 2010 FALL 2025 TAKE-HOME FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
Let A, B, and C be subsets of a set U.
Then $(\overline{A} \cup B) \cap (\overline{B} \cup C) \subseteq \overline{A} \cup C$.

(2) Prove or disprove:
Let p be a prime number. Then $p+2$ prime is necessary and sufficient for p to be odd.

(3) Prove or disprove:
For each positive integer n,
the integer $n^5 + 5n^3 - 6n$ is a multiple of 5.

(4) Prove or disprove:
For a positive real number x, there is a natural number n such that $(n+1)x > 1 \geq nx$.

(5) Prove or disprove:
If a monotonic sequence $\{x_n\}$ has a bounded subsequence, then $\{x_n\}$ converges.

(6) Prove or disprove:
Consider a Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$, and a number $K > 0$. Suppose that there is a subsequence $\{x_n\}_{n\in R}$, all of whose terms are smaller than K, and another subsequence $\{x_n\}_{n\in S}$, all of whose terms are larger than K. Then $\lim_{n\to\infty} = K$.