MATH 2010-2 FALL 2024 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
Let A, B, and C be subsets of a set U.
Then $(\overline{A} \cap B) \cup (\overline{B} \cap C) \subseteq \overline{A} \cap C$.

(2) Prove or disprove:
The inequality $4x^2 + 9y^2 \geq 12xy$ holds
for all real numbers x and y.

(3) Prove or disprove:
For each positive integer n,
the integer $n^5 + 5n^2 + 6n$ is a multiple of 5.

(4) Prove or disprove:
For a real number $x > -1$ and a positive integer n,
$(1 + x)^n \geq 1 + nx$.

(5) Prove or disprove:
If a monotonic sequence $\{x_n\}$ has a bounded
subsequence, then $\{x_n\}$ converges.

(6) Prove or disprove:
The series
$$\sum_{n=5}^{\infty} \frac{2n}{n^2 - 4n - 1}$$
converges.