MATH 2010 SPRING 2026 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
The compound statements $(P \to Q) \wedge P$ and $P \wedge Q$ are logically equivalent.

(2) Prove or disprove:
For each natural number n, the integer $n^5 + 4n$ is a multiple of 5.

(3) Prove or disprove:
For real numbers x and y, the inequality

$$|x - y| < |x + y|$$

holds iff x and y are both positive.

(4) Prove or disprove:
The function

$$f : \mathbb{Q} \to \mathbb{R}; x \mapsto \frac{x + \sqrt{2}}{x\sqrt{2} + 1}$$

is injective.

(5) Prove or disprove:
Let E and F be bounded nonempty subsets of $\mathbb{R}$.
Suppose that $\forall\, x \in E,\ \exists\, y \in F\,.\, x < y\,.$
Then $\sup E \leq \sup F$.

(6) Prove or disprove:
A sequence $\{x_n\}_{n \in N}$ satisfying

$$\forall\, \varepsilon > 0\,,\ \exists\, M\,.\, \forall\, M \leq n\,,\ |x_{n+1} - x_n| < \varepsilon$$

is a Cauchy sequence.