MATH 2010 SPRING 2026 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Determine the limit of the sequence

$$\left\{ \frac{n^4 - 3n^2 + 7}{2n^4 + 5n + 10} \right\}_{n \in \mathbb{N}},$$

justifying your answer.

(2) Suppose that $x_n \to L$ for some sequence x_n. Suppose that

$$\exists\, R \neq 1 \,.\, \forall\, n \in \mathbb{N}\,,\ x_{n+1} = Rx_n.$$

(a) Show that $Rx_n \to RL$.
(b) Show that $Rx_n \to L$.
(c) Show that $L = 0$.

(3) Give a proof, by induction, of the following

Proposition. Suppose that for each natural number n, there is a continuous function $f_n \colon \mathbb{R} \to \mathbb{R}$. Then for each natural number n, the function $(f_n \circ f_{n-1} \circ \ldots \circ f_1 \circ f_0)(x)$ is continuous.