MATH 2010-1 SPRING 2025 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
Let A, B, and C be subsets of a set U.
Then $(C \smallsetminus A) \cap (A \smallsetminus B) = (A \cap C) \smallsetminus B$.

(2) Prove or disprove:
For each natural number n which is not a multiple of 5, the integer $n^4 - 1$ is a multiple of 5.

(3) Prove or disprove:
For real numbers x, y with $x + y > -1$,
$$(1 + x + y)^n \geq 1 + nx + ny$$
for each natural number n.

(4) Prove or disprove:
The function
$$f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}; (p, q) \mapsto \frac{(1 - p + pq)(1 + q - pq)}{1 + p^2 + q^2}$$
is invertible.

(5) Prove or disprove:
Suppose that $\{x_n\}$ and $\{y_n\}$ are convergent sequences of real numbers with $x_n \leq y_n$ for all n.
Then $\lim_{n \to \infty} x_n \leq \lim_{n \to \infty} y_n$.

(6) Prove or disprove:
The series
$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$
diverges.