

### MATH 2010 SPRING 2025 GRADED HOMEWORK #3

*Write clearly, on separate paper. All questions carry equal weight.*

- (1) Determine the limit of the sequence

$$\left\{ \frac{6n^3 - 4n^2 \cos(n)}{3n^3 + 4n + 1} \right\}_{n \in \mathbb{N}},$$

justifying your answer.

- (2) Suppose that  $f: \mathbb{N} \rightarrow \mathbb{N}$  is injective, with  $f(0) \neq 0$ . Determine the limit of the sequence  $\{1/f(n)\}_{n \in \mathbb{N}}$ , justifying your answer.
- (3) Give a proof, by induction, of the following

**Proposition.** Suppose that  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are continuous functions. Then, for each natural number  $n$ , the function  $(g \circ f)^n: x \mapsto (g(f(x)))^n$  is continuous.