

M2010-1 SPRING 2025 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Consider the functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = x^4 \quad \text{and} \quad g(x) = \sqrt{2x + 1}.$$

Find the formulas for $f \circ g$ and $g \circ f$.

- (2) Find the inverse of the function $f: \mathbb{R} \setminus \{\frac{1}{2}\} \rightarrow \mathbb{R} \setminus \{\frac{1}{2}\}$ with

$$f(x) = \frac{x - 1}{2x - 1}.$$

- (3) Prove or disprove the following:

Claim. Suppose that $A \cup B$ is a countably infinite set. Then A and B are countably infinite sets.

- (4) Prove, by induction, that $n^3 < 3^n$ for positive integers $n > 3$.