MATH 2010 FALL 2026 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Determine the limit of the sequence

$$\left\{\frac{n^3+7}{3n^3-4n+6}\right\}_{n\in\mathbb{N}},$$

justifying your answer.

(2) Give a careful proof of the following:

Proposition. Suppose that a sequence $\{x_n\}_{n\in U}$ of positive terms is unbounded. Then there is a subsequence $\{x_n\}_{n\in S}$ of $\{x_n\}_{n\in U}$ such that $\{x_n^{-1}\}_{n\in S}$ converges to 0.

(3) Consider the sequence $\{(-1)^n\}_{n\in\mathbb{N}}$.

(a) Determine the limit of the subsequence $\{(-1)^n\}_{n\in 2\mathbb{N}}$.

(b) Determine the limit of the subsequence $\{(-1)^n\}_{n\in 1+2\mathbb{N}}$.

(c) Explain why the divergence of $\{(-1)^n\}_{n\in\mathbb{N}}$ follows directly from your answers to (a) and (b).