MATH 504 F'14 SUPPLEMENTARY EXERCISES A

(1) Let R be a commutative, unital ring. Let P be an ideal of R. Show that the following conditions are equivalent:
   (a) The quotient ring R/P is an integral domain;
   (b) If elements x and y of R lie outside P, then the product xy also lies outside P.
   (c) For all elements x, y of R, if xy lies in P, then at least one of x and y lies in P.

   [If these conditions hold, the ideal P is described as being *prime*.]

(2) Suppose that $f\colon R \to S$ is a unital ring homomorphism between commutative, unital rings. If Q is a prime ideal of S, show that $f^{-1}(Q)$ is a prime ideal of R.

   [The set of prime ideals of a ring R is called the (*prime*) *spectrum* $\mathrm{Spec}(R)$ of R. Then the function

$$\mathrm{Spec}(S) \to \mathrm{Spec}(R); Q \mapsto f^{-1}(Q)$$

   is written as $\mathrm{Spec}(f)$.]