

MATH 505 SPRING 2015 PRACTICE MIDTERM

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Let S be a unital ring. Let F be the free right S -module over a set X , and let G be the free right S -module over a set Y . Show that $F \otimes G$ is the free right S -module over the cartesian product $X \times Y$ of the sets X and Y .

- (2) Let n be a positive integer. Let

$$0 \rightarrow A_0 \rightarrow A_1 \rightarrow \dots \rightarrow A_{n-1} \rightarrow A_n \rightarrow 0$$

be an exact sequence of finite-dimensional real vector spaces. Show that

$$\sum_{k=0}^n (-1)^k \dim A_k = 0.$$

- (3) Show that the embedding $j: \mathbb{Z} \hookrightarrow \mathbb{Q}$ is an epimorphism in the category of unital ring homomorphisms.
- (4) Suppose that p is a prime number, and that n is a positive integer. Determine the Galois group of $\text{GF}(p^n)$ over $\text{GF}(p)$.