# MATH 504 FALL 2014 GRADED HOMEWORK #0

*Write clearly, on separate paper. All questions carry equal weight.*
*You will receive credit for your three best answers.*

(1) A subset $S$ of $\mathbb{Z}$ is said to be *summary* if the two conditions
   (i) $n \leq m$,
   (ii) $\exists\, r \in S\,.\, m = n + r$
   are equivalent for each ordered pair $(n, m)$ of elements of $S$.
   (a) Show that $\mathbb{N}$ is summary.
   (b) If $S$ contains $\mathbb{N}$ as a proper subset, show that $S$ is not summary.

(2) Let $a$ and $b$ be positive integers. Show that their greatest common divisor is their smallest positive integral linear combination.

(3) Show that for all natural numbers $n$, the integers $3n - 1$ and $4n - 1$ are coprime.

(4) Let $n$ be a positive integer. A positive integer $d$ is said to be a *unitary divisor* of $n$ if $d$ divides $n$, and $\gcd(d, n/d) = 1$. In this case, $n$ is said to be a *unitary multiple* of $d$.
   (a) Determine the unitary divisors of 72 and 1200.
   (b) Determine the least common unitary multiple of 18 and 45.
   (c) Show that there is no least common unitary multiple of 3 and 9.