MATH 307B SPRING 2012 PRACTICE TEST #5

Write clearly. Show your working. All questions carry equal weight.

(1) Suppose $\mathbf{x}_1 = [1 \quad 0 \quad 1]^T$ and $\mathbf{x}_2 = [1 \quad 2 \quad 1]^T$. Find an orthogonal basis for the span of the set $\{\mathbf{x}_1, \mathbf{x}_2\}$.

(2) Suppose $\mathbf{y} = [1 \quad 1 \quad 2]^T$, $\mathbf{u}_1 = [-2 \quad 1 \quad 0]^T$, and $\mathbf{u}_2 = [-1 \quad -2 \quad 1]^T$.

   (a) Show that the set $\{\mathbf{u}_1, \mathbf{u}_2\}$ is orthogonal.
   (b) Find the orthogonal projection of the vector $\mathbf{y}$ onto the span of the set $\{\mathbf{u}_1, \mathbf{u}_2\}$.

(3) Find the slope m and y-intercept c of the straight line $y = mx + c$ which gives the least squares fit to the data points $(1, 2)$, $(2, 1)$, and $(3, 2)$.

(4) For the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix},$$

find a diagonal matrix D and orthogonal matrix P such that $A = PDP^{-1}$.

(5) Suppose that $A = PDP^{-1}$ with a diagonal matrix D and orthogonal matrix P. Let λ be an eigenvalue of A of multiplicity k, so that λ appears precisely k times on the diagonal of D. Explain why the dimension of the eigenspace of A corresponding to λ is k.