MATH 307B SPRING 2012 PRACTICE TEST #4

Write clearly. All questions carry equal weight.

(1) Compute the real eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}.$$

(2) Compute the determinant

$$\begin{vmatrix} 2 & -1 & 3 & 0 \\ a & b & c & 0 \\ 0 & 0 & 0 & 2 \\ a & b-1 & c & 0 \end{vmatrix}.$$

(3) Consider the following statements, and determine whether each is true or false. You do not need to justify your claim. In each case, A represents a (5×5)-matrix.
   (a) $\det 5A = 5 \det A$.
   (b) If the set of (real) roots of the characteristic equation of A is $\{-2, 2, 3\}$, then A is definitely not diagonalizable.
   (c) If the set of (real) roots of the characteristic equation of A is $\{-2, -1, 1, 2, 3\}$, then A is diagonalizable.
   (d) If A is singular, but not diagonalizable, then 0 is not an eigenvalue of A.

(4) For the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

find a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$.

(5) Compute the matrix of the linear transformation

$$T : \mathbb{P}_2 \to \mathbb{P}_3; p(t) \mapsto \int_{x=0}^{x=t} p(x)\, dx$$

with respect to the standard bases $\mathcal{B} = \{1, t, t^2\}$ and $\mathcal{C} = \{1, t, t^2, t^3\}$.