

MATH 307B SPRING 2012 PRACTICE TEST #3

Write clearly. All questions carry equal weight.

(1) Consider the transformation $T : \mathbb{P}_3 \rightarrow \mathbb{R}^3; p(t) \mapsto \begin{bmatrix} p(0) \\ p'(1) \\ p''(2) \end{bmatrix}$. Is it linear? Justify your answer.

(2) Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 2 & 8 & 1 \\ 0 & 4 & 4 & 1 \end{bmatrix}.$$

Determine each of the following:

- (a) A basis for $\text{Col } A$;
 - (b) A basis for $\text{Nul } A$;
 - (c) $\text{rank } A$.
- (3) Determine the coordinate vector of the polynomial $p(t) = 1 - t + t^2$ with respect to the ordered basis $\mathcal{B} = \{1, 1 + t, (1 + t)^2\}$ of \mathbb{P}_2 .
- (4) Suppose $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$. Determine the change-of-coordinates matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$.
- (5) A (5×7) -matrix A has rank 4. Determine $\dim \text{Nul } A$, $\dim \text{Row } A$, and $\text{rank } A^T$.