

MATH 307B SPRING 2012 PRACTICE TEST #2

Write clearly. All questions carry equal weight.

- (1) Consider the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- (a) Give a set of vectors that span the null space of A .
(b) Is the linear transformation $L_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3; \mathbf{x} \mapsto A\mathbf{x}$ injective (“1-1”)? Justify your answer.

- (2) Consider the matrices

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

For each of the following, either compute the indicated matrix, or state that it is not defined:

- (a) $A + 2B$; (b) $A^T - 2B$; (c) AB ; (d) AC .

- (3) Consider the linear transformation $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ with

$$T(x_1, x_2, x_3, x_4) = (x_2 - x_3, 2x_1, x_2 + x_4).$$

- (a) Determine the standard matrix A of T (so that $T = L_A$).
(b) Is T injective (“1-1”)? Justify your answer.
(c) Is T surjective (“onto”)? Justify your answer.

- (4) For each of the following matrices, either determine the inverse, or explain why the matrix is singular:

$$(a) \begin{bmatrix} 7 & -1 & -4 \\ -4 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad (b) \begin{bmatrix} 7 & -1 & -4 \\ -4 & -1 & 2 \\ 11 & 0 & -6 \end{bmatrix}$$

- (5) For each of the following sets W of vectors of the form shown, either find a set of elements that spans W , or give an example to show that W is *not* a vector space.

$$(a) \begin{bmatrix} a \\ a - b + c \\ 2a - c \\ c + d \end{bmatrix} \quad (b) \begin{bmatrix} a \\ a - b \\ b - 1 \\ d + 1 \end{bmatrix}$$