

MATH 307B SPRING 2012 PRACTICE TEST #1

Write clearly. All questions carry equal weight.

- (1) Following are the **augmented** matrices of systems of equations. In each case, determine whether the corresponding system has a unique solution, infinitely many solutions, or no solution.

(a)

$$\begin{bmatrix} 1 & 9 & 0 & 0 & 7 & 0 \\ 0 & 0 & 1 & 1 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & 0 & 5 & -2 & 0 \\ 2 & 4 & 1 & 5 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

- (2) Give the solution to the following system in parametric vector form:

$$\begin{aligned} 2x_1 + x_2 - x_3 + x_4 &= -3 \\ x_1 - x_2 + x_3 + x_4 &= 4 \\ 4x_1 - x_2 + x_3 + 3x_4 &= 5 \end{aligned}$$

- (3) Suppose that \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 are four-dimensional column vectors. Does

$$\text{Span}\{\mathbf{u}_1 - \mathbf{u}_2, \mathbf{u}_2 - \mathbf{u}_3, \mathbf{u}_1\}$$

contain the vector \mathbf{u}_3 ? Justify your answer.

- (4) Are the columns of the following matrix linearly independent?

$$\begin{bmatrix} 1 & -4 & 3 & 3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

If they are dependent, exhibit a non-trivial dependence relation between them.

- (5) Let A be a 4×3 matrix, and let \mathbf{b} be a 4-dimensional column vector. Suppose that the matrix equation $A\mathbf{x} = \mathbf{b}$ has a unique solution. Show that the columns of A are linearly independent.