Write clearly. All questions carry equal weight.

(1) Consider the matrix
$$A = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$
   (a) Give a set of vectors that span the null space of A.
   (b) Is the linear transformation $L_A \colon \mathbb{R}^3 \to \mathbb{R}^3; \mathbf{x} \mapsto A\mathbf{x}$ injective ("1-1")? Justify your answer.

(2) Consider the matrices
$$A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

   For each of the following, either compute the indicated matrix, or state that it is not defined:
   (a) $A + 2B$;   (b) $A^T - 2B$;   (c) AB;   (d) AC.

(3) Consider the linear transformation $T \colon \mathbb{R}^4 \to \mathbb{R}^3$ with
$$T(x_1, x_2, x_3, x_4) = (x_2 - x_3, 2x_1, x_2 + x_4).$$
   (a) Determine the standard matrix A of T (so that $T = L_A$).
   (b) Is T injective ("1-1")? Justify your answer.
   (c) Is T surjective ("onto")? Justify your answer.

(4) For each of the following matrices, either determine the inverse, or explain why the matrix is singular:
$$\text{(a)} \begin{bmatrix} 7 & -1 & -4 \\ -4 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 7 & -1 & -4 \\ -4 & 1 & 2 \\ 11 & 0 & -6 \end{bmatrix}$$

(5) The (3×3)-matrices A, B, and C are such that ABC is invertible. Show that CAB is invertible.