MATH 302 SPRING 2014 PRACTICE FINAL

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

1. Let x be an element of a unital ring R. If $x^7 = 0$, show that $1 + x$ is an invertible element of R.
2. Let I be an ideal of a ring R.
   (a) If I is contained in an ideal J of R, show that J/I is an ideal of R/I.
   (b) If K is an ideal of R/I, show that there is a unique ideal J of R, containing I, such that $K = J/I$.
3. The graphs of two cubic functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ each pass through the points $(0, 10), (1, 12), (2, 8)$ and $(3, 1)$. Prove that $f = g$.
4. Find a real number a such that $\mathbb{Q}(\sqrt{3}, \sqrt{7}) = \mathbb{Q}(a)$. Justify your answer.
5. Consider the evaluation homomorphism
$$\theta \colon \mathbb{Q}[X] \to \mathbb{R}; f(X) \mapsto f\left(\sqrt{1+\sqrt{2}}\right).$$
Find a monic polynomial $p(X)$ in $\mathbb{Q}[X]$ such that $\operatorname{Ker}\theta = p(X)\mathbb{Q}[X]$.
6. Find all 3 monic irreducible polynomials of degree 2 over the field of order 3.