

## MATH 302 SPRING 2012 PRACTICE TEST #2

*Write clearly. Box or underline your final answers to computational questions.  
All questions carry equal weight.*

1. Show that the field of fractions of the integral domain  $\mathbb{Z}[i]$  is  $\mathbb{Q}[i]$ .
2. Show that  $\mathbb{Q}(\sqrt{5}, \sqrt{7}) = \mathbb{Q}(\sqrt{5} + \sqrt{7})$ .
3. Let  $J$  be the ideal  $(X^2 + X + 1)\mathbb{Z}/5[X]$  of the ring  $\mathbb{Z}/5[X]$  of polynomials over the 5-element field  $\mathbb{Z}/5$ . Find the inverse of the coset  $X + J$  in the quotient field  $\mathbb{Z}/5[X]/J$ .
4. Find a monic polynomial  $p(X)$  in  $\mathbb{Q}[X]$  such that the quotient field  $\mathbb{Q}[X]/p(X)\mathbb{Q}[X]$  is isomorphic to the field  $\mathbb{Q}(\sqrt{1 + i\sqrt{3}})$ .