MATH 302 SPRING 2012 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

1. Show that the set
$$\{0, 2, 4, 6, 8, 10, 12\}$$
of residues modulo 14 forms a field.

2. (a) Show that $7\mathbb{Z}/_{21}$ is an ideal of the ring $\mathbb{Z}/_{21}$ of integers modulo 21.
   (b) Show that $(\mathbb{Z}/_{21})\big/(7\mathbb{Z}/_{21}) \cong \mathbb{Z}/_7$.

3. Is the polynomial X^2+X+1 irreducible over the field of integers $\mathbb{Z}/_7$ modulo 7? Justify your answer.

4. Consider the function
$$f\colon \mathbb{Z}/_5^* \to \mathbb{Z}/_5^*; x \mapsto x^{-1}.$$
Find a polynomial $p(X)$ in $\mathbb{Z}/_5[X]$ for which f is the restriction (to $\mathbb{Z}/_5^*$) of the corresponding polynomial function.