

MATH 301 SPRING 2017 PRACTICE TEST #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

(1) Set

$$G = \left\{ \begin{bmatrix} p & q \\ r & s \end{bmatrix} \mid p, s \in \mathbb{Z}, q \in 2\mathbb{Z}, r \in 3\mathbb{Z}, ps - qr = 1 \right\}.$$

Show that G is a subgroup of the group of invertible 2×2 real matrices under (the usual) matrix multiplication.

(2) Let a and b be positive integers. Consider the set

$\mathcal{M} = \{G \mid a\mathbb{Z} \subseteq G, b\mathbb{Z} \subseteq G, \text{ and } G \text{ is a subgroup of } (\mathbb{Z}, +, 0)\}$
of subgroups of $(\mathbb{Z}, +, 0)$. Prove that

$$\bigcap_{G \in \mathcal{M}} G = a\mathbb{Z} + b\mathbb{Z}.$$

(3) Show that a subgroup N of a group G is normal if and only if each right coset Nx of N with an element x of G is equal to the left coset yN with some element y of G .

(4) Let N be a normal subgroup of a group G . Let H be a subgroup of G . Show that NH is a subgroup of G .