

MATH 301 SPRING 2017 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

- (1) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a . Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
 - (a) Show that G contains σ_{ma} for each positive integer m .
 - (b) Show that G contains σ_{ma} for each integer m .
 - (c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .
- (2) Let α be a permutation of a set X . Give a careful proof to show that $\alpha^{m+n} = \alpha^m \circ \alpha^n$ for all integers m and n .
- (3) Let β , and $\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$, be permutations of a finite set X . Show that
$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$
- (4) Give a careful proof to show that $\log_{13} 23$ is irrational.