

MATH 301B SPRING 2016 PRACTICE TEST #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

(1) Set

$$G = \left\{ \begin{bmatrix} p & q \\ r & s \end{bmatrix} \mid p, q, s \in \mathbb{Z}, r \in 3\mathbb{Z}, ps - qr = 1 \right\}.$$

Show that G is a subgroup of the group of invertible 2×2 real matrices under (the usual) matrix multiplication.

- (2) Show that a subgroup N of a group G is normal if and only if each right coset Nx of N with an element x of G is equal to the left coset yN with some element y of G .
- (3) Let M and N be normal subgroups of a group G . Show that MN is a normal subgroup of G .
- (4) Let $d_k d_{k-1} \dots d_1 d_0$ be the decimal expansion of a positive integer n , so that

$$n = \sum_{j=0}^k d_j 10^j$$

with $0 \leq d_j < 10$. Show that 3 divides n if and only if

$$d_k + d_{k-1} + \dots + d_1 + d_0 \equiv 0 \pmod{3}.$$