

MATH 301B SPRING 2016 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions.

All questions carry equal weight.

- (1) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a . Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
- (a) Show that G contains σ_{ma} for each positive integer m .
 - (b) Show that G contains σ_{ma} for each integer m .
 - (c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .

- (2) Let $f : X \rightarrow Y; x \mapsto f(x)$ be a function.
- (a) Show that there is a subset Y' of Y such that

$$g : X \rightarrow Y'; x \mapsto f(x)$$

is surjective.

- (b) Show that there is a subset X' of X such that

$$h : X' \rightarrow Y'; x \mapsto f(x)$$

is bijective.

- (3) Let β , and $\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$, be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\ \beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)) .$$

- (4) Show that $\sqrt[5]{3}$ is irrational.