

MATH 301B SPRING 2011 PRACTICE TEST #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Determine the group of units $(\mathbb{Z}/16, \cdot, 1)^*$ of the monoid $(\mathbb{Z}/16, \cdot, 1)$ of integers modulo 16 under multiplication.
- (2) Let H and K be subgroups of a group G . Suppose that
 $\forall h_1 \in H, k_1 \in K, \exists h_2 \in H, k_2 \in K. k_1 h_1 = h_2 k_2.$
Show that $L = \{hk \mid h \in H, k \in K\}$ is a subgroup of G .
- (3) Let G be a group, with subgroups H and K . If $|H| = 98$ and $|K| = 75$, show that $H \cap K$ is trivial.
- (4) Let $f : (X, \cdot, e_X) \rightarrow (Y, \cdot, e_Y)$ be an invertible group homomorphism. Show that $f^{-1} : (Y, \cdot, e_Y) \rightarrow (X, \cdot, e_X)$ is a group homomorphism.