MATH 301B SPRING 2011 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

(1) Let a be an integer. Prove that there are uniquely specified integers q and e such that

$$a = 5q + e$$

with $|e| < 3$.

(2) Let $f : X \to Y; x \mapsto f(x)$ be a function.
   (a) Show that there is a subset Y' of Y such that

$$g : X \to Y'; x \mapsto f(x)$$

   is surjective.
   (b) Show that there is a subset X' of X such that

$$h : X' \to Y'; x \mapsto f(x)$$

   is bijective.

(3) Consider the set

$$S = \{\sigma_n \colon \mathbb{R} \to \mathbb{R}; x \mapsto n + x \mid 3 \Big| n \in \mathbb{N}\}$$

of real shifts.
   (a) Show that S is a monoid of functions.
   (b) Show that S is not a group of permutations.

(4) Show that

$$(a_1\ a_2\ \ldots\ a_{r-1}\ a_r) \circ (b_1\ b_2\ \ldots\ b_{s-1}\ b_s) = (b_1\ b_2\ \ldots\ b_{s-1}\ b_s) \circ (a_1\ a_2\ \ldots\ a_{r-1}\ a_r)$$

for disjoint cycles $(a_1\ a_2\ \ldots\ a_r)$ and $(b_1\ b_2\ \ldots\ b_s)$.