

MATH 301A FALL 2013 PRACTICE TEST #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Set

$$G = \left\{ \begin{bmatrix} p & q \\ r & s \end{bmatrix} \mid p, q, s \in \mathbb{Z}, r \in 3\mathbb{Z}, ps - qr = 1 \right\}.$$

Show that G is a subgroup of the group of invertible 2×2 real matrices under (the usual) matrix multiplication.

- (2) Let (G, \cdot, e) be a finite group, and let p be an odd prime number. Consider the equation

$$x^p = e$$

for an element x of G . Show that the number of solutions x in G is odd.

- (3) Let M and N be normal subgroups of a group G . Show that MN is a normal subgroup of G .
- (4) For $d = 7 \times 23 = 161$, consider the monoid $(\mathbb{Z}/161\mathbb{Z}, \cdot, 1)$ of integers modulo d under multiplication. Determine the order of the group of units $(\mathbb{Z}/161\mathbb{Z}, \cdot, 1)^*$ of $(\mathbb{Z}/161\mathbb{Z}, \cdot, 1)$.