

MATH 301A FALL 2013 PRACTICE TEST #1

Write clearly. All questions carry equal weight.
You will receive credit for your three best answers.

- (1) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a . Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
- (a) Show that G contains σ_{ma} for each positive integer m .
 - (b) Show that G contains σ_{ma} for each integer m .
 - (c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .

- (2) Let X be an infinite set. A function $f : X \rightarrow X$ is said to be *almost identical* if the set

$$\{x \in X \mid x \neq f(x)\}$$

of elements x of X , differing from their image $f(x)$ under f , is finite. Let F be the subset of X^X consisting of the almost identical functions. Show that F is a monoid of functions.

- (3) Let β , and $\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$, be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$

- (4) Show that $\log_{10} 3$ is irrational.