MATH 301A FALL 2010 PRACTICE TEST #1

Write clearly. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Write $\sigma_a : \mathbb{R} \to \mathbb{R}; x \mapsto a + x$ for the shift by a real number a. Suppose that a group G of permutations of $\mathbb{R}$ contains σ_a and σ_b for real numbers a and b.
   (a) Show that G contains σ_{ma} for each positive integer m.
   (b) Show that G contains σ_{ma} for each integer m.
   (c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b.

(2) Let β, and $\alpha = (x_1 \ x_2 \ \ldots \ x_{r-1} \ x_r)$, be permutations of a finite set X. Show that
$$\beta \circ \alpha \circ \beta^{-1} = \big(\ \beta(x_1) \ \beta(x_2) \ \ldots \ \beta(x_{r-1}) \ \beta(x_r) \big) \,.$$

(3) Let $f : X \to Y$ be a function. Prove or disprove the existence of a function $g : Y \to X$ such that $f = f \circ g \circ f$.

(4) Consider the set X of positive real numbers. Define a relation R on X by
$$x \ R \ y \quad \text{if and only if} \quad |x - y| < 1 \,.$$
Prove or disprove that R is an equivalence relation on X.