MATH 2010 SPRING 2026 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] Let $\{E_i \mid i \in I\}$ be an indexed family of nonempty subsets of a bounded subset E of $\mathbb{R}$. Show that

$$\sup \bigcup_{i \in I} E_i = \sup\{\sup E_i \mid i \in I\}\,.$$

(2) [4pts.] Show that the function

$$f\colon [-2,1] \to \mathbb{R}; x \mapsto x^3 - 2x + 4\sin 3x$$

is bounded.

(3) [5pts.] Consider a real number x with $|x| < 1$. Prove that

$$\left|\frac{1-x^{r+1}}{1-x}\right| \leq \frac{1-|x|^{r+1}}{1-|x|}$$

for $r \in \mathbb{N}$.