MATH 201 SPRING 2024 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
For subsets $A = \{x \in U \mid P(x)\}$ and
$B = \{x \in U \mid Q(x)\}$ of a set U, we have
$$\overline{A} \cup B = \{x \in U \mid P(x) \to Q(x)\}.$$

(2) Prove or disprove:
For each positive integer n,
the integer $n^5 + 4n$ is a multiple of 5.

(3) Prove or disprove:
For subsets A and B of $\mathbb{R}$, if $a < b$ for all $a \in A$ and $b \in B$, then $\sup A \leq \inf B$.

(4) Prove or disprove:
If a decreasing sequence $\{x_n\}$ has a bounded subsequence, then the sequence $\{x_n\}$ converges.

(5) Show that the series
$$\sum_{n=2}^{\infty} \frac{2n+1}{3n^2(n+1)^2}$$
converges, and find the limit.

(6) Prove or disprove: The series
$$\sum_{n=2}^{\infty} \frac{1}{n \log n^2}$$
converges.