MATH 201 SPRING 2024 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] Consider the subset $E = \{x \mid 2^x < 3\}$ of $\mathbb{Q}$.
 (a) Show that E is a bounded above in $\mathbb{Q}$.
 (b) Show that no rational number can be the least upper bound of E.

(2) [4pts.] Show that the function $f \colon [-3, 2] \to \mathbb{R}; x \mapsto x^3 - 2x + 12$ is bounded.

(3) [5pts.] Suppose $\varepsilon > 0$.
 (a) Show that $(1 + \varepsilon)^n \geq 1 + n\varepsilon$ for each natural number n.
 (b) Show that the sequence
$$\left\{ \frac{1}{(1 + \varepsilon)^n} \right\}_{n \in \mathbb{N}}$$
 is decreasing and bounded below.
 (c) Show that
$$\lim_{n \to \infty} \frac{1}{(1 + \varepsilon)^n} = 0 \,.$$