MATH 201 SPRING 2024 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.

(1) Find the inverse of the function $f\colon \mathbb{R} \smallsetminus \{1\} \to \mathbb{R} \smallsetminus \{2\}$ with
$$f(x) = \sqrt[3]{\frac{8x - 7}{x - 1}}\,.$$

(2) Prove the following result by induction:

Proposition. Let $\{X_r \mid r \in \mathbb{N}\}$ be a family of countable sets. Then for all natural numbers n, the set
$$\bigcup_{r=0}^{n}(X_{r+1} \smallsetminus X_r)$$
is countable.

(3) Prove by induction: $\forall\, 4 \le n \in \mathbb{N}$, $n^3 \le (n+1)!$.