

MATH 201 SPRING 2023 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

- (1) Prove or disprove:

For sets A, B, C , we have

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

- (2) Prove or disprove:

For each positive integer n ,
the integer $n^5 - n$ is a multiple of 5.

- (3) Prove or disprove:

For bounded subsets A and B of \mathbb{R} , if $a \leq b$ for all
 $a \in A$ and $b \in B$, then $\sup A \leq \inf B$.

- (4) Prove or disprove:

If an increasing sequence $\{x_n\}$ has a bounded
subsequence, then the sequence $\{x_n\}$ converges.

- (5) Show that the series

$$\sum_{n=2}^{\infty} \frac{2n+1}{n^2(n+1)^2}$$

converges, and find the limit.

- (6) Prove or disprove:

(a) The series

$$\sum_{n=10}^{\infty} \frac{1}{n^{10} \log n}$$

converges;

(b) The series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n^2}$$

converges.