MATH 201 SPRING 2023 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Determine the limit of the sequence

$$\left\{ \frac{4n^3 - 10n^2 + 5}{2n^3 - 3n - 2} \right\}_{n \in \mathbb{N}},$$

justifying your answer.

(2) Suppose $0 < x \in \mathbb{R}$.
   (a) Prove: $\forall\ n \in \mathbb{N}\,,\ (1+x)^n \geq 1 + nx$.
   (b) Prove: $\lim_{n\to\infty}(1+x)^{-n} = 0$.

(3) Give a proof, by induction, of the following

Proposition. For each natural number n,
the function $1 + x + x^2 + \ldots + x^n$ is continuous.