

## MATH 201 SPRING 2023 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.  
You will receive credit for your three best answers.*

- (1) Consider the functions  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = x^2 \quad \text{and} \quad g(x) = \sqrt{x^2 + 1}.$$

Find the formulas for  $f \circ g$  and  $g \circ f$ .

- (2) Find the inverse of the function  $f: \mathbb{R} \setminus \{-2\} \rightarrow \mathbb{R} \setminus \{2\}$  with

$$f(x) = \frac{2x + 3}{x + 2}.$$

- (3) Prove or disprove:

**Proposition.** For sets  $A$  and  $B$ , if  $A \setminus B$  and  $A$  are countable, then  $B$  is countable.

- (4) Prove by induction:  $\forall 3 < n \in \mathbb{N}, 2n + n^2 \leq n!$