MATH 201 SPRING 2021 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:

Let A, B, and C be subsets of a set U.
Then $(C \smallsetminus A) \cup (A \smallsetminus B) = (A \cup C) \smallsetminus B$.

(2) Prove or disprove:

The inequality $4x^2 + 9y^2 \geq 6xy$ holds
for all real numbers x and y.

(3) Prove or disprove:

For each positive integer n,
the integer $n^5 + 5n^2 - 6n$ is a multiple of 5.

(4) Prove or disprove:

The function

$$f \colon \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}; (p, q) \mapsto \frac{(1 - p + pq)(1 + q - pq)}{1 + p^2 + q^2}$$

is invertible.

(5) Prove or disprove:

Suppose that $\{x_n\}$ and $\{y_n\}$ are convergent sequences
of real numbers with $x_n \leq y_n$ for all n.
Then $\lim_{n \to \infty} x_n \leq \lim_{n \to \infty} y_n$.

(6) Prove or disprove:

If $\{x_n\}_{n \in S}$ and $\{x_n\}_{n \in S'}$ are convergent subsequences
of a sequence $\{x_n\}_{n \in U}$, with respective limits L and
L', then $L = L'$.