MATH 201 SPRING 2021 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Using only the definition of convergence to a limit for a sequence, give a careful formal proof of the following:

Prop. Suppose that a sequence $\{x_n\}_{n\in\mathbb{N}}$ of nonzero real numbers converges to a nonzero limit x. Then

$$\lim_{n\to\infty}[2x_n - 4] = 2x - 4.$$

(2) Give a formal proof by induction of the following:

Prop. For each positive integer i, let $\{(x_i)_n\}_{n\in\mathbb{N}}$ be a sequence that converges to a limit L_i. Then

$$\lim_{n\to\infty}\Big[(x_1)_n + (x_2)_n + \ldots + (x_r)_n\Big] = L_1 + L_2 + \ldots + L_r$$

for each positive integer r.

You may quote the following lemma without proving it:

Lemma. If $\lim_{n\to\infty} y_n = Y$ and $\lim_{n\to\infty} z_n = Z$, then $\lim_{n\to\infty}[y_n + z_n] = Y + Z$.

(3) Prove that the sequence

$$\left\{\frac{(n^2+1)\sin(n^3-5n)}{n^3+4}\right\}_{0<n\in\mathbb{N}}$$

converges to zero.