MATH 201 FALL 2019 FINAL GRADED HOMEWORK

Write clearly, on separate paper, then scan to PDF for submission.
Alternatively, create and submit a PDF from LaTeX, Word, or other software.
All questions carry equal weight.

(1) Give a proof, by induction, of the following:

Proposition. For each positive integer n, the number $n^5 - 11n$ is divisible by 5.

(2) Prove or disprove the following statement:

Let $\{a_n\}_{n \in U}$ and $\{b_n\}_{n \in U}$ be sequences.
If $\lim b_n = 0$, then the sequence $\{a_n b_n\}_{n \in U}$ converges.

(3) (a) Show that the sequence

$$\sum_{n=2}^{\infty} \frac{1}{n(n-1)} \tag{1}$$

converges, and find its limit.

(b) By comparison incorporating the series (1), show that the series

$$\sum_{n=0}^{\infty} \frac{1}{n!} = 2 + \sum_{n=2}^{\infty} \frac{1}{n!}$$

converges.

(c) What upper bound for Euler's number e do you obtain from your comparison?