

MATH 201 SPRING 2019 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

- (1) For a real number $x > -1$ and a positive integer n , prove that $(1 + x)^n \geq 1 + nx$.
- (2) Prove or disprove:
For each positive integer n ,
the integer $n^5 - n$ is a multiple of 5.
- (3) Prove or disprove:
For bounded subsets A and B of \mathbb{R} , if $a \leq b$ for all $a \in A$ and $b \in B$, then $\sup A \leq \inf B$.
- (4) Prove or disprove:
If a monotonic sequence $\{x_n\}$ has a bounded subsequence, then the sequence $\{x_n\}$ converges.
- (5) Show that the series

$$\sum_{n=2}^{\infty} \frac{2n+1}{n^2(n+1)^2}$$

converges, and find the limit.

- (6) Prove or disprove:
(a) The series

$$\sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$$

converges;

- (b) The series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n^2}$$

converges.