

## MATH 201 SPRING 2019 PRACTICE TEST #2

*Write clearly, on separate paper.*

- (1) [5pts.] Let  $A$  be a totally ordered set with the least upper bound property. Let  $E$  be a nonempty subset that is bounded below. Let  $L$  be the set of lower bounds for  $E$ .
- (a) Show that  $L$  is nonempty.
  - (b) Show that  $L$  is bounded above.
  - (c) Show that  $\sup L$  is a lower bound for  $E$ .
  - (d) Show that  $\sup L$  is the greatest lower bound of  $E$ .
- (2) [5pts.] For elements  $x, y$  of an ordered field  $F$ , with  $0 < x < y$ , show that
- $$\forall 0 < n \in \mathbb{N}, \forall 0 < r \leq n, x^{n-r+1}y^{r-1} < x^{n-r}y^r.$$
- Conclude that
- $$\forall 0 < n \in \mathbb{N}, x^n < y^n.$$
- (3) [4pts.] Find a number  $M$  such that  $|x^5 - 2x^2 + 3x| \leq M$  for all  $-1 \leq x \leq 2$ . Justify your claim.