

MATH 201 SPRING 2019 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

- (1) Give a careful, detailed proof of the following

Proposition. The sequence $\{2^n\}_{n \in \mathbb{N}}$ is unbounded.

Your proof should use the Archimedean Property and Russell's Paradox.

- (2) Working directly from the basic definition of convergence to a limit, show that $\lim_{n \rightarrow \infty} x_n = x$ and $\lim_{n \rightarrow \infty} y_n = y$ together imply that $\lim_{n \rightarrow \infty} (2x_n - 3y_n) = 2x - 3y$.
- (3) Give a proof, by induction, of the following

Proposition. For $0 < n \in \mathbb{N}$, suppose that the functions $f_1, \dots, f_n: \mathbb{R} \rightarrow \mathbb{R}$ are continuous. Then

$$f_n \circ f_{n-1} \circ \dots \circ f_2 \circ f_1$$

is continuous.