

MATH 201 SPRING 2018 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

- (1) Let A , B , and C be subsets of a set U . Prove or disprove:

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

- (2) (a) Show that

$$\forall 0 < x \in \mathbb{R}, \forall 1 \leq n \in \mathbb{N}, 1 + nx \leq (1 + x)^n.$$

- (b) Consider $1 < r \in \mathbb{R}$. Show that the sequence $\{r^n\}$ is unbounded.

- (3) Prove or disprove:

For each positive integer n ,
the equality

$$\sum_{k=1}^n 2^{-k} + 2^{-n} = 1$$

holds.

- (4) Prove or disprove:

If the sequence $\{x_n^8\}$ converges, then the sequence $\{x_n\}$ also converges.

- (5) Suppose that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. Give a careful, direct proof that $\{x_n y_n\}$ is a Cauchy sequence.

- (6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

does not converge.