MATH 201 SPRING 2018 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Give a careful formal proof of the following:

> **Prop.** Suppose that a sequence $\{x_n\}_{n\in\mathbb{N}}$ of nonzero real numbers converges to a nonzero limit x. Then:
>
> (a) There is a positive real number R such that
> $$\forall\, n \in \mathbb{N}\,,\ |x_n| \geq R\,.$$
> (b) The sequence $\{x_n^{-1}\}_{n\in\mathbb{N}}$ converges to x^{-1}.

(2) Give a formal proof by induction of the following:

> **Prop.** Let $f_i\colon \mathbb{R} \to \mathbb{R}$ be a continuous function for each positive integer i. Then the function $f_1 + \ldots + f_n$ is continuous for each positive integer n.

You may quote the following lemma without proving it:

> **Lemma.** If $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ are continuous functions, then $f + g$ is continuous.

(3) Consider the sequence $\{x_n\}_{n\in\mathbb{N}}$ with $x_n = (-1)^n$ for $n \in \mathbb{N}$. Give a direct proof that $\{x_n\}$ is not a Cauchy sequence.