MATH 201A SPRING 2015 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight. You will receive credit for your three best answers.

- (1) Show that $n < 2^n$ for each natural number n.
- (2) Find the inverse of the function $f: \mathbb{R} \smallsetminus \{\frac{1}{2}\} \to \mathbb{R} \smallsetminus \{\frac{5}{2}\}$ with

$$f(x) = \frac{5x - 2}{2x - 1} \,.$$

(3) Prove or disprove the following:

Claim. Suppose that X is a countably infinite subset of an uncountable set Y. Then the set $Y \setminus X$ is uncountable.

(4) Prove or disprove the following:

Claim. Suppose that X and Y are uncountable sets. Then if X is a proper subset of Y, the set $Y \setminus X$ is uncountable.